Microstrip superconducting quantum interference device radio-frequency amplifier: Effects of negative feedback on input impedance

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We present the results of measurements of the scattering parameters of microstrip amplifiers (MSAs) based on the dc superconducting quantum interference device. The amplifier input impedance is poorly matched to typical transmission line impedances, resulting in high input return loss around −2 dB. We show that negative feedback can lower the MSA input impedance to achieve a robust match to 50 Ω. In the presence of capacitive and inductive feedback, the input return loss of the MSA can be reduced below −10 dB, opening the door to the practical use of the MSA for a variety of demanding applications. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3114419]

The dc superconducting quantum interference device (dc SQUID) has been used as an ultralow noise amplifier at frequencies of up to several gigahertz.[1–6] In the most common mode of SQUID operation, the input signal is applied across the two ends of a superconducting coil that is separated from the SQUID washer by a thin insulating layer and tightly coupled to the SQUID loop.[7] At signal frequencies above several tens of megahertz, however, the parasitic capacitance between the input coil and the SQUID washer results in a drastic decrease in the useful gain of the device. This problem can be circumvented by coupling the input signal between one end of the coil and the washer of the SQUID.[4] The input circuit now acts like a microstrip resonator: when the input coil is driven at a frequency close to the $\lambda/2$ standing-wave resonance of the microstrip, there is a current enhancement in the input circuit by a factor of Q (the microstrip quality factor), leading to enhanced gain and reduced noise in the SQUID. We refer to a SQUID operated in this fashion as a *microstrip SQUID amplifier* (MSA).

The noise temperature of a MSA cooled to 20 mK in a dilution refrigerator has been measured to be 50 mK at a frequency of 540 MHz, an order of magnitude lower than the best semiconductor amplifiers available.[8] The unsurpassed noise performance of the MSA is quite attractive for a variety of basic science applications, including readout of the dark-matter axion detector,[9] low-noise postamplification of the single electron transistor,[10] and low-noise readout of superconducting flux qubits.[11] However, the practical usefulness of the MSA has been quite limited. This is due in part to the fact that the input impedance of the MSA is poorly matched to typical transmission line impedances, complicating efforts to cascade multiple MSA stages and thereby increase gain. To optimize the performance of the MSA and extend its practical usefulness, it is necessary to understand the (generally complex) input impedance of the MSA and to develop robust techniques for matching the MSA input to 50 Ω. In this letter we present measurements of the input and output impedance of MSAs and describe the use of negative feedback to match the MSA input impedance to 50 Ω.

The input impedance z_i of the MSA is that of a resonant microstrip, substantially modified by the presence of the SQUID. The dynamic resistance $\Re$ of the SQUID is a complicated nonlinear function of the current and flux biases. Hilbert and Clarke[12] demonstrated that for optimal SQUID bias and in the presence of feedback, the impedance coupled into the input circuit is R, the shunt resistance per junction; at different bias points, $\Re$ can be substantially different from R. In a simplified model of the MSA, one expects the input circuit to look like the SQUID dynamic impedance transformed up by the turns ratio n of the input coil: $z_i \approx n^2 R$. For an input circuit with an $n=10$ turn coil and for a shunt resistance of $R=5\ \Omega$, we expect $z_i \approx 500\ \Omega$. Indeed, Kinion and Clarke[5] measured $z_i \approx 500\ \Omega$ for a MSA with a resonant frequency of 500 MHz, in agreement with the simple model presented above.

For input impedance of order 500 Ω, one expects to find input return loss $|S_{11}|^2$ of around −2 dB. The high levels of reflected power are problematic as they could degrade the noise performance of the previous amplification stage or result in the unwanted back action in the case of qubit readout. For reliable integration of the MSA into a 50 Ω environment, we require input return loss of order −10 dB, corresponding to input impedance in the range from 25 to 100 Ω if we neglect the imaginary part of the impedance.

A convenient way to modify the input impedance of an amplifier is to feed back a part of the output signal to the input. Hilbert and Clarke[1] studied feedback in the context of SQUID amplifiers using a simple lumped circuit model. Feedback was accomplished by injecting a current from the SQUID output to the input via a feedback capacitor. It was found that positive feedback increased the gain on resonance, while negative feedback suppressed SQUID gain. At the same time, the input impedance was increased by positive feedback and decreased by negative feedback. To model the MSA in the presence of capacitive feedback, we consider the device as a transimpedance amplifier with forward transimpedance $a=-jQMV_\Phi$ and feedback transadmittance $f=j\omega C_{fb}$ on resonance, where $V_\Phi \equiv \partial V/\partial\Phi$ is the SQUID transfer function. We find a loop gain

$$T=\omega QMV_\Phi C_{fb}. \qquad (1)$$

In the presence of feedback, the MSA gain A and input and output impedances $Z_{i,o}$ are reduced by a factor $1+T$ with respect to their values a and $z_{i,o}$ in the absence of feedback,

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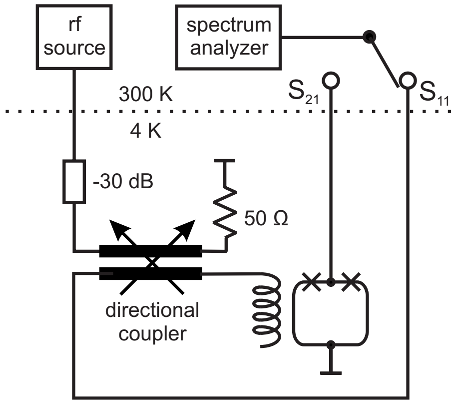

FIG. 1. Experimental configuration for measurement of MSA scattering parameters. A directional coupler is used to measure the input return loss. The input reflected power and output power are coupled to a spectrum analyzer via a coaxial relay.

$$A = \frac{a}{1+T}; \quad Z_{i,o} = \frac{z_{i,o}}{1+T}. \tag{2}$$

Finally, negative feedback improves amplifier nonlinearity, yielding a reduction in total harmonic distortion by a factor of T. For example, in order to reduce the input impedance of a MSA operating at 500 MHz from 500 to 100 Ω, we require a loop gain of $T=4$. For a device with $Q=5$, $M=3.5$ nH, and $V_\Phi=200$ μV/Φ_0, we find that a feedback capacitance $C_{fb}=1$ pF reduces the input return loss from −2 to −10 dB with an acceptable decrease in gain of around 3.5 dB.

We have made measurements of the gain and input and output impedance of MSAs with and without capacitive feedback. The SQUIDs had inner and outer washer dimensions of 200 μm and 1 mm, respectively, an estimated SQUID inductance of $L_S \approx 350$ pH, typical values for the SQUID critical current of $I_c \approx 8-10$ μA, and resistance of $R \approx 15-20$ Ω. Measured values of V_Φ are typically 200 μV/Φ_0. The microstrip resonator had a width of 5 μm and a length of 12 mm, forming a nine-turn coil on top of the SQUID washer. The SQUIDs incorporated Nb–AlO_x–Nb tunnel junctions and Pd resistors and were fabricated using conventional sputter deposition and photolithographic techniques.

The performance of several amplifiers was measured at 4.2 K using current and flux biases I_B and I_Φ set to achieve maximum gain; Fig. 1 shows the measurement configuration. A cold 30 dB attenuator prevented noise from the room temperature signal generator from saturating the SQUID and presented a 50 Ω impedance to both the input coaxial line and the microstrip. For the same reasons, a cold 2 dB attenuator coupled the SQUID output to a low-noise amplifier at room temperature via a coaxial line. The gain of the entire system excluding the SQUID was calibrated by connecting together the input and output attenuators, and the SQUID gain measurements were referred to the baseline so obtained. In all measurements, the washer of the SQUID was grounded and the counterelectrode was at the potential of the output signal. The input and output return losses were measured with a cold directional coupler mounted close to the SQUID. A coaxial relay switched the output of the SQUID or the directional coupler to the spectrum analyzer. This was to avoid having to manually change connectors on the probe, which could have changed the bias flux in the SQUID due to vibrations of the probe. Figure 2 shows a typical measurement of the forward gain $|S_{21}|^2$, input return loss $|S_{11}|^2$, and output return loss $|S_{22}|^2$ of a MSA operated with and without a feedback capacitance of $C_{fb}=1$ pF. It is seen that capacitive feedback reduces the input return loss, with a modest reduction in forward gain. There is also a reduction in the output impedance.

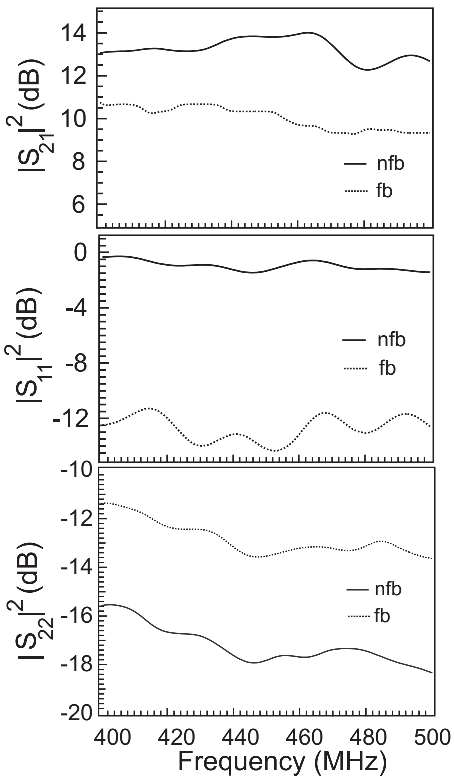

FIG. 2. Forward gain $|S_{21}|^2$, input return loss $|S_{11}|^2$, and output return loss $|S_{22}|^2$ of a MSA operated without (nfb) and with (fb) a 1 pF feedback capacitance. The feedback capacitance was provided by a surface mount ceramic capacitor placed on the same printed circuit board as the SQUID, in close proximity to the SQUID chip.

At higher frequencies in the range of 1–2 GHz, the inductance of the leads used to ground the SQUID washer causes the SQUID washer to develop a nonzero rf potential due to the finite output current. In this case, coupling from the SQUID washer to the microstrip input circuit provides sufficient feedback to reduce the MSA input impedance below 100 Ω even in the absence of an explicit capacitance from output to input. In Fig. 3 we show $|S_{21}|^2$ and $|S_{11}|^2$

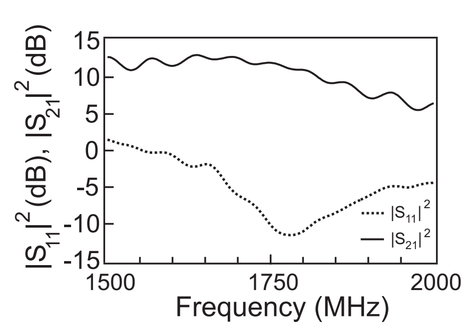

FIG. 3. Forward gain $|S_{21}|^2$ and input return loss $|S_{11}|^2$ for a MSA with feedback due to the inductance of the leads grounding the SQUID washer.

versus frequency for a MSA with a resonant frequency of 1500 MHz and an estimated inductance between the washer and ground of 2 nH. This MSA had a smaller washer size of $10\times 200\ \mu m^2$, a shunt resistance of $R=10\ \Omega$, and a 13-turn microstrip resonator. While one would naively expect an input impedance of the order of $n^2R\approx 1700\ \Omega$, the low input reflection indicates that Z_i is of the order of $100\ \Omega$ due to the inductive feedback.

In conclusion, we have shown that the input match of a MSA to a $50\ \Omega$ source can be improved by applying negative feedback either in the form of a capacitor connected between the input and output of the MSA or via a small inductor between the SQUID washer and ground. This negative feedback will reduce the available gain of the MSA by about 3–4 dB and will decrease the output impedance by a factor of about 1.5, which are both tolerable. The realization of MSAs that are well matched to transmission line impedances opens the door to the robust cascading of multiple MSA stages and to use of the MSA in a variety of demanding practical applications.

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